Code No: R1921044 **R19** SET - 1

II B. Tech I Semester Regular Examinations, March - 2021
RANDOM VARIABLES AND STOCHASTIC PROCESSES
(Electronics and Communication Engineering)

Time: 3 hours Max. Marks: 75

Answer any **FIVE** Questions each Question from each unit
All Questions carry **Equal** Marks
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1 a) Define conditional probability distribution function and write he properties [8M]

b) A random variable X is defined by [7M]

$$X(i)=\begin{cases}-2 & i\le -2\\ i & -2<i\le 1\\ 1 & 1<i\le 4\\ 6 & 4<i\end{cases}$$

Show, by a sketch, the value x into which the values of i are mapped by x.
What type of random variable is X?

Or

2 a) Given that a random variable X has the following possible values, state if X is discrete, continuous or mixed [8M]

i. {-20<x<-5}
ii. {10,12<x<=14,15,17)
iii. {-10 for s>2 and 5 for s<=2, where 1<s<=6}
iv. {4,3.1,1, -2}

b) Suppose height to the bottom of clouds is a Gaussian random variable for which ax=4000m and σx=1000m. A person bets that cloud height tomorrow will fall in the set A={1000m<X≤3000m} while a second person bets that height will be satisfied by B={2000m<X≤4200m}.A third person bets they are both correct. Find the probability that each person will win the bet. [7M]

3 a) The random variable X has characteristics function ϕ_X(w)=[a/a-jw]^N for a>0 and N=1,2,3…… Show that $\bar{X}$=N/a, $\bar{X}^2$N(N+1)/a^2, and σ_x^2=N/a^2. [8M]

b) Find mean and variance of Gaussian random variable? [7M]

Or

4 a) A random variable X is uniformly distributed on the interval (-5,15).Another random variable Y=$e^{(-\frac{X}{5})}$ is formed. Find E[Y]. [8M]

b) A Gaussian voltage random variable X has a mean value ax=0 and σ^2x=9. The voltage X is applied to a square-law, full wave diode detector with a transfer characteristics Y=5X^2. Find the mean value of the output voltage Y. [7M]

5 a) Random variable X and Y have the joint density [8M]

$F_{X,Y}$(x,y)={ 1/24 0<x<6 and 0<y<4
0 elsewhere.

What is the expected value of the function g(X,Y)=(XY)^2?

b) Two statistically independent random variable X and Y have mean values $\bar{X}$ =E[X]=2 and E[Y]=4. They have second moments $\bar{X}^{\,2}$=E[X^2]=8 and E[Y^2]=25. Find i) the mean value ii) the second moment iii) the variance of the random variable W=3X-Y. [7M]

Or
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6 a) For the two random variable X and Y: [8M]

$F_{X,Y}(x,y)=0.15\delta(x+1)\delta(y)+0.1\delta(x)\delta(y)+0.1\delta(x)\delta(y-2)+0.4\delta(x-1)\delta(y+2)$
$+0.2\delta(x-1)\delta(y-1)+0.5\delta(x-1)\delta(y-3)$

Find: i) the correlation, ii) the covariance, iii) the correlation coefficient of X and Y and iv) are X and Y either uncorrelated or orthogonal?

b) Gaussian random variable X_1 and X_2 for which $\bar{X}_1$=2,σ_{X1}^2=9,$\bar{X}_2$=-1, σ_X^2=4 and C_{X1X2}=-3 are transformed to new random variable Y_1 and Y_2 according to Y_1=-X_1+X_2 ,Y_2=-2X_1-3X_2. Find [7M]

i)σ_{Y1}^2 ii)σ_{Y2}^2 iii) C_{Y1Y2}.

7 a) Let X (t) be a stationary continuous random process that is differentiable. Denote its time derivative by $\dot{X}(t)$. Show that $E\left(\dot{X}(t)\right]=0$. [8M]

b) Given the random process by X(t)=A cos(w_0t) + B sin(w_0t) [7M]

Where w_0 is a constant, and A and B are uncorrelated zero mean random variables having different density functions but the same variance ,show that X(t) is wide sense stationary but not strictly stationary.

Or

8 a) A random process is defined by X(t) = A, where A is a continuous random variable uniformly distributed on (0, 1). Determine the form of the sample functions, classify the process [8M]

b) Define ergodic random proven? Explain with example. [7M]

9 a) Drive the Wiener-Khintchine relation. [10M]

b) What is Mean value of System Response for Random Signal Response of Linear Systems. [5M]

Or

10 A Random signal X(t) of PSD of $\frac{N_0}{2}$ is applied on an LTI system having impulse response $h(t)$. If $Y(t)$is output, find (i) $E[Y^2(t)]$ (ii) $R_{XY}(\tau)$ (iii) $R_{YX}(\tau)$ (iv) $R_{YY}(\tau)$. [15M]

Code No: R1921044 **R19** SET - 2

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RANDOM VARIABLES AND STOCHASTIC PROCESSES
(Electronics and Communication Engineering)

Time: 3 hours Max. Marks: 75

Answer any **FIVE** Questions each Question from each unit
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1 a) Define Random variable? List out the properties of Distribution Function [8M]

b) A random variable X is known to be Poisson with b=0 [7M]
Plot the density and distribution functions for this random variable.
What is the probability of event $\{0 \le X \le 5\}$

Or

2 a) Explain Gaussian random variable with neat sketches? [8M]

b) For the gaussian density function of ax=0 and σx=1, show that [7M]
$\int_{-\infty}^{\infty} x f x(x) dx$=ax

3 a) A random variable X is uniformly distributed on the interval $(-\pi/2, \pi/2)$. X is transformed to the new random variable Y = T (X) = a tan (X), where a> 0. Find the probability density function of Y. [8M]

b) Show that characteristics function of a random variable having the binomial density function is Φ(w)=[1-p+pe^{jw}]^N. [7M]

Or

4 a) A random variable X has $\bar{X}$=-3, $\bar{X}^2$=11 and σ_X^2=2. For a new random variable Y=2X-3, [8M]
Find: i)$\bar{Y}$ ii)$\bar{Y}^2$ iii)σ_Y^2.

b) For the poisson random variable show that mean and variance is same [7M]

5 a) Two gaussian random variables X and Y have variances σ_X^2 =9 and σ_Y^2 =4 respectively and correlation coefficient ρ. It is known that a coordinate rotation by angle -π/8 results in new random variable Y_1 and Y_2 that are uncorrelated. What is ρ? [8M]

b) Two random variables X and Y are defined by $\bar{X}$=0,$\bar{Y}$=-1, $\bar{X}^2$=2,$\bar{Y}^2$=4 and R_{XY}=-2. Two random variable W and U are: W=2X+Y, U=-X-3Y. Find $\bar{W}$,$\bar{U}$, $\bar{W}^2$,$\bar{U}^2$,R_{WU},σ_X^2,σ_Y^2. [7M]

Or

6 a) Random variable X and Y have the joint density function [8M]

$F_{X,Y}(x,y)$={ $(x+y)^2/40$ -1<x<1 and -3<y<3
0 elsewhere.

i)find all the second order moments of X and Y
ii) what are variances of X and Y.
iii) What is the correlation coefficient?

b) Two gaussian random variables X and Y are variances σ_X^2 =4 and σ_Y^2 =9 respectively and correlation coefficient ρ. It is known that a coordinate rotation by angle -π/4 results in new random variable Y_1 and Y_2 that are uncorrelated. What is ρ? [7M]

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7 a) Write the properties of Autocorrelation Function of Random Process [6M]

b) A Gaussian random process is known to be a WSS process with mean $\bar{X} = 4$ and $R_{XX}(\tau)$=$25e^{-3|\tau|}$ where $\tau = \frac{|t_k - t_i|}{2}$ and i,k=1,2. Find joint Gaussian density function? [9M]

Or

8 a) What is wide-sense stationary random process and explain with example [8M]

b) Define Random Process and classify it. [7M]

9 a) A random process had the power density spectrum [8M]

$$S(\omega) = \frac{6\omega^2}{1 + \omega^4}$$

Find the average power in the process

b) Assume X(t) is a wide sense stationary process with non zero mean value. show that [7M]

$$s_{xx}(\omega) = 2\pi\bar{X}^2\delta(\omega) + \int_{-\infty}^{\infty} C_{xx}(\tau)e^{-j\omega\tau}d\tau$$

where $C_{xx}(\tau)$ is the auto covariance function of X(t).

Or

10 a) Derive the relationship between Cross-Power Density Spectrum and Cross-Correlation Function. [8M]

b) Explain Band pass Processes with Properties. [7M]

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1 a) Define Density Function? List out the properties of Density Function [8M]

b) Gaussian random voltages X for which $a_X = 0$ and $\sigma_X = 4.2V$ appears across a 100-Ω resistor with power rating of 0.25W. What is the probability that the voltage will cause an instantaneous power that exceeds the resistor's rating? [7M]

Or

2 a) Define Poisson Random variable? What type of applications it will suitable and give the relationship between Poisson and Binomial Random variable. [8M]

b) For the Gaussian density function of ax=0 and σx=1, show that [7M]

$$\int_{-\infty}^{\infty} (x - \mathrm{ax})^2 \, fx(x)dx = \sigma x^2$$

3 a) Explain about Transformation of random variable [8M]

b) For the binomial density function, show that E[X] = Np and variance = Np(1-p) [7M]

Or

4 a) Find the mean, variance from moment generation function of uniform distribution? [8M]

b) A random variable X can have values -4, -1,2,3,4 each with probability 1/5.Find: i) the density function ii) the mean iii) the variance of the random variable $Y=3X^2$. [7M]

5 a) Define Marginal density function? Find the Marginal density functions of below joint density function. [8M]

$$f_{XY} = \frac{1}{12} u(x)u(y)e^{-x/3}e^{-y/4}$$

b) Two random variables having joint characteristic function $\emptyset_{XY}(\omega_1, \omega_2)=\exp(-2\omega_1^2-8\omega_2^2)$. Find moment's m_{10}, mo_1, m_{11}? [7M]

Or

6 a) Find the density function of W=X+Y, where the densities of X and Y are assumed to be: $f_x(x)=4u(x)e^{-4x}$; $f_y(y)=5u(y)e^{-5y.}$ [10M]

b) Joint Sample Space has three elements (1, 1), (2, 2), and (3, 3) with probabilities 0.4, 0.3, 0.3 respectively then draw the Joint Distribution Function diagram [5M]

7 a) Let two random processes X(t) and Y(t) be defined by [8M]

$$X(t) = A \cos \omega_0 t + B \sin \omega_0 t$$
$$Y(t) = B \cos \omega_0 t - A \sin \omega_0 t$$

where A and B are random variables and ω_0 is a constant. Assume A and B are uncorrelated, zero mean random variables with same variance. Find the cross correlation function $R_{XY}(t,t+\tau)$.

b) Write the properties of Cross correlation Function of Random Processes. [7M]

Or
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8 a) What is strict-sense stationary random process and explain with example. [8M]

b) What is Cross- Correlation Function and explain its Properties . [7M]

9 a) Write the properties of power density spectrum.

b) If X(t) is a stationary process, find the power spectrum of $Y(t) = A_0 + B_0 X(t)$ in term of the power spectrum of X(t) if A_0 and B_0 are real constants. [8M]

Or [7M]

10 a) Explain Band-Limited Processes with Properties. [7M]

b) If X(t) is band limited process such that $S_{xx}(\omega) = 0, when\ |\omega| > \sigma$, prove that $2[R_{xx}(0) - R_{xx}(\tau)] \leq \sigma^2\tau^2 R_{xx}(0)$. [8M]

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1 a) Define Random variable? Write the conditions for a function to be random variable [6M]

b) A random voltage can have any value defined by the set 'S' = {a≤ s ≤ b}. A quantizer, divides S into 6 equal-sized contiguous subsets and generates random variable X having values {-4, -2, 0, 2, 4, 6}. Each value of X is earned to the midpoint of the subset of 'S' from which it is mapped [9M]

i) Sketch the sample space and the mapping to the line that defines the values of X

ii) Find a and b?

Or

2 a) Explain about Gaussian random variable [8M]

b) A Gaussian random variable X has $a_X = 2, and \sigma_X = 2$ [7M]

i. Find $P\{X > 1.0\}$

ii. Find $P\{X \le -1.0\}$

3 a) A random variable X has a probability density [8M]

$$f_X(x) = \begin{cases} (1/2)\cos(x) & -\pi/2 < x < \pi/2 \\ 0 & elsewhere\ in\ x. \end{cases}$$

Find the mean value of the function on $g(X) = 4X^2$

b) Let X be a Poisson random variable then find out its mean and variance [7M]

Or

4 a) Find the expected value of the function $g(X) = X^3$ where X is a random variable defined by the density [8M]

$$f_X(x) = \left(\frac{1}{2}\right) u(x) \exp(-x/2).$$

b) State and prove Chebchev's inequality? [7M]

5 a) For two random variables X and Y [8M]

$$f_{X,Y}(x, y) = 0.15\delta(x+1)\delta(y) + 0.1\delta(x)\delta(y) + 0.1\delta(x)$$

$$\delta(y-2) + 0.4\delta(x-1)\delta(y+2) +$$

$$0.2\delta(x-1)\delta(y-1) + 0.5\delta(x-1)\delta(y-3).$$

Find the correlation coefficients of X and Y

b) Gaussian random variables X and Y have first and second order moments m_{10}=-1.1, m_{20}=1.16, m_{01}=1.5, m_{02} =2.89, R_{XY}=-1.724 find C_{XY} and ρ? [7M]

Or
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6 a) Define random variables V and W by [8M]

$$V = X + aY$$
$$W = X - aY$$

Where a is real number and X and Y random variables, Determine a in terms of X and Y such V and W are orthogonal?

b) Gaussian random variable X_1 and X_2 for which $\bar{X}_1$=2,σ_{X1}^2=9,$\bar{X}_2$=-1, σ_X^2=4 and C_{X1X2}=-3 are transformed to new random variable Y_1 and Y_2 according to Y_1=-X_1+X_2 ,Y_2=-2X_1-3X_2. Find [7M]

i)$\bar{X}^2$ ii) $\bar{X}^2$ iii)ρ_{X1X2} iv)σ_{Y1}^2

7 a) Given that the autocorrelation function for a stationary Ergodic process with no period components is [8M]

$$R_{xx}(\tau) = 25 + \frac{4}{1 + 6\tau^2}$$

Find the mean and variance of process X(t)?

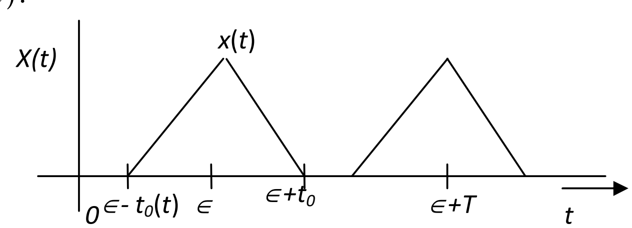
b) Give the random process by [7M]

$$X(t) = A\cos(w_0 t) + B\sin(w_0 t)$$

Where $w_0$is a constant, and A and B are uncorrelated zero mean random variables having different density functions but the same variance ,show that X(t) is wide sense stationary but not strictly stationary

Or

8 A random process $X(t)$ has periodic sample functions as show in figure ; where B, T and $4t_0 \le T$ are constants but $\in$ is a random variable uniformly distributed on the interval (0, T). Find first order density function and distribution function of $X(t)$. 15M



9 a) Derive the relationship between power spectrum and autocorrelation [8M]

b) The autocorrelation function of a random process X(t) [7M]

$$R_{xx}(\tau) = 3 + 2\exp(-4\tau^2)$$

i. Find the power spectrum of X(t)
ii. What is the average power in X(t)?

Or

10 a) Explain Power Density Spectrum of Response Characteristics of LTI System Response [8M]

b) Explain Narrowband Processes with Properties. [7M]