

II B. Tech I Semester Supplementary Examinations, September - 2021
RANDOM VARIABLES AND STOCHASTIC PROCESSES
 (Electronics and Communication Engineering)

Time: 3 hours

Max. Marks: 75

Answer any **FIVE** Questions each Question from each unit
 All Questions carry **Equal** Marks

- 1 a) A random variable has a PDF given by [8M]

$$f_X(x) = \begin{cases} 0; & -\infty < x \leq -1 \\ 0.5 + 0.5x; & -1 < x < 1 \\ 1; & 1 \leq x < \infty \end{cases}$$

- (i) Find the probability that $X = \frac{1}{4}$

- (ii) Find $P\left(X > \frac{3}{4}\right)$

- b) Show that $\int_{-\infty}^{\infty} f_X(x) dx = 1$. [7M]

Or

- 2 a) Find the mean and mean-square value of uniform random variable. [7M]

- b) A random variable X has a PDF of the form $f_X(x) = \frac{1}{4}[u(x+2) - u(x-2)]$. [8M]

- (i) Plot $f_X(x)$

- (ii) Find $F_X(x)$.

- 3 a) If the density function of a continuous random variable is given by [7M]

$$f_X(x) = \frac{1}{2}e^{-|x|} \quad \text{Find the MGF.}$$

- b) Determine the standard deviation of uniform random variable. [8M]

Or

- 4 a) Given that X is a uniform random variable distributed over $(0,1)$. Find $f_Y(y)$, if [7M]
 $Y = X^3$.

- b) Define $\Phi_X(\omega)$ and $M_X(s)$. List the properties of these functions. [8M]

- 5 a) Show that $\Phi_{XY}(\omega_1, \omega_2) = \Phi_X(\omega_1)\Phi_Y(\omega_2)$, if X and Y are statistically [7M]
 independent random variables.

- b) If $Z = X + Y$, find $f_Z(z)$. Given that X and Y are statistically independent [8M]
 random variables.

Or

- 6 a) Two random variables X and Y have the joint characteristic function [7M]
 $\Phi_{XY}(\omega_1, \omega_2) = \exp(-2\omega_1^2 - 8\omega_2^2)$. Show that X and Y are both zero-mean
 random variables.

- b) List all the properties of joint cumulative distribution function. [8M]

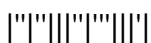
- 7 a) Give the classification of random processes. [7M]

- b) A stationary random process has an autocorrelation function given by [8M]

$$R(\tau) = \frac{25\tau^2 + 36}{6.25\tau^2 + 4}$$

Find the mean value, mean-square value and variance of the process.

Or



- 8 a) If $X(t) = A$, where A is a random variable, prove that $X(t)$ is not mean-ergodic. [7M]
b) For a zero-mean stationary random process, show that $K_{XX}(\tau) = R_{XX}(\tau)$. [8M]
- 9 a) Derive the Wiener-Khinchin relation for ACF and PSD. [7M]
b) If $X(t)$ is a WSS process, show that $S_{XY}(\omega) = S_{XX}(\omega)H^*(\omega)$. [8M]
- Or
- 10 a) The autocorrelation function of the random telegraph signal process is given by $R(\tau) = a^2 \exp(-2b|\tau|)$. Determine the power spectral density of the telegraph signal. [7M]
b) If $X(t)$ is a WSS process, show that $R_{XY}(\tau) = R_{XX}(\tau) * h(-\tau)$. [8M]

